Reply to "Comment on 'Majoron emitting neutrinoless double beta decay in the electroweak chiral gauge extensions' "

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We demonstrate that in the process of deducing the constraint on the electroweak mixing angle θ_W in our paper [1], we have indeed been working with three mass scales while implementing (331) model.

In their comment Montero et al. point out that there must exist at least three different mass scales for the scalar vacuum expectation values if majoron like scheme is implemented within the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model. We agree with the authors of the comment and take this opportunity to correct a typo in our paper [1] as well as demonstrate that we have in fact been working with three mass scales in the process of deducing the constraint on the electroweak mixing angle θ_W.

We re-examine the generation of masses in $G_{331} \equiv SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model. To implement the symmetry breaking hierarchy,

$$G_{331} \rightarrow G_{321} \rightarrow SU(3)_C \otimes U(1)_{\text{em}}, \tag{1}$$

a scalar sector composed of SU(3) symmetric sextet of scalar fields,

$$S = \begin{pmatrix} \sigma_1^0 & h_2^+ & h_1^- \\ h_2^+ & H_1^{++} & \sigma_2^0 \\ h_1^- & \sigma_2^0 & H_2^{--} \end{pmatrix} \sim (\mathbf{1}, \mathbf{6_s^*}, 0), \tag{2}$$

and $SU(3)_L$ triplets,

$$\eta \sim (\mathbf{1}, \mathbf{3}, 0), \quad \rho \sim (\mathbf{1}, \mathbf{3}, +1), \quad \chi \sim (\mathbf{1}, \mathbf{3}, -1), \tag{3}$$

with the vacuum structure

$$\langle \eta \rangle = (v_\eta, 0, 0), \quad \langle \rho \rangle = (0, v_\rho, 0), \quad \langle \chi \rangle = (0, 0, v_\chi), \tag{4}$$

and

$$\langle S \rangle = \begin{pmatrix} v_{\sigma_1} & 0 & 0 \\ 0 & 0 & v_{\sigma_2} \\ 0 & v_{\sigma_2} & 0 \end{pmatrix}, \tag{5}$$

is introduced. One may note that the introduction of sextet is not essential for the symmetry breaking. In case the Eq. (4) holds, the gauge symmetry breaks to $SU(3)_C \otimes U(1)_{\text{em}}$. However it results in an antisymmetric mass matrix for charged leptons with one eigenvalue being zero and other two equal in magnitude, for three generations. A VEV of the sextet is needed to produce a realistic mass matrix of the charged leptons [2].

At this point we would like to correct a typo in our paper due to which it appeared as if only two mass scales have been used. The first line after Eq. (21) of our paper [1] should read

"For $v_{\sigma_1} = 0$, notice that even if $v_\eta \approx v_\rho \approx \sqrt{2} v_{\sigma_2} \equiv v_1$, the VEV $v_\chi \equiv v_2$ must be large enough in order to leave the new gauge bosons sufficiently heavy to keep consistency with low energy phenomenology. "

instead of

"Notice that even if $v_\eta \approx v_\rho \approx v_{\sigma_1} \approx v_{\sigma_2} \equiv v_1$ where v_1 denotes the usual vacuum expectation value for the Higgs boson of the standard model, the VEV $v_\chi \equiv v_2$ must be large enough in order to leave the new gauge bosons sufficiently heavy to keep consistency with low energy phenomenology. "

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We recall here that the scalar field σ_1^0 transforms as a triplet and σ_2^0 transforms as a doublet of the subgroup SU(2). The Masses of Charged vector bosons are

$$M_W^2 = \frac{g^2}{2}\left(v_\eta^2 + v_\rho^2 + 2v_{\sigma_2}^2 + 2v_{\sigma_1}^2\right) \tag{6}$$

and

$$M_Z^2 = \frac{g^2}{2c_W^2}(v_\eta^2 + v_\rho^2 + 2v_{\sigma_2}^2 + 4v_{\sigma_1}^2) \tag{7}$$

An extra overall factor of $\frac{1}{2}$ and different coefficients for $v_{\sigma_1}^2$ in Eqs. (2) and (3) of Ref. [3] as compared to our Eqs. (6) and (7) above is due to a difference in their choice of vacuum structure for SU(3)$_L$ triplets and symmetric sextet of scalar fields in comparison with ours. Now with the approximation $v_\eta \approx v_\rho \approx \sqrt{2}v_{\sigma_2} \equiv v_1$ we obtain

$$M_W^2 = \frac{g^2}{2}\left(3v_1^2 + 2v_{\sigma_1}^2\right), \tag{8}$$

$$M_Z^2 = \frac{g^2}{2c_W^2}\left(3v_1^2 + 4v_{\sigma_1}^2\right) \tag{9}$$

The order of magnitude of v_{σ_1} can be estimated from the experimental constraint that is the value of ρ-parameter: $\rho = 0.9998 \pm 0.0008$. Using Eqs. (8) and (9), we obtain

$$0.9998 = \frac{1 + \frac{2}{3}r}{1 + \frac{4}{3}r} \quad ; \text{ giving } \quad \sqrt{r} = \frac{v_{\sigma_1}}{v_1} = 0.0173.$$

For the choice $v_1^2 = \left(\frac{246}{\sqrt{6}}\right)^2 \text{GeV}^2 \approx (100)^2 \text{GeV}^2$, we get $v_{\sigma_1} \le 1.73$ GeV. The following treatment leading to the constraint on the electroweak mixing angle θ_W deals with a very special choice that is $v_{\sigma_1} = 0$, and $v_2 \gg v_1$. For this particular case Eq. (8) gives $M_W^2 = \frac{3}{2}g^2 v_1^2$ and $\rho = 1$.

Using the dimensionless parameters

$$A \equiv \left(\frac{v_1}{v_2}\right)^2 \tag{10}$$

and

$$t \equiv \frac{g'}{g} \tag{11}$$

where g and g' are the SU(3)$_L$ and U(1)$_X$ gauge coupling constants the mass matrix for the neutral gauge bosons in the $\{W_\mu^3, W_\mu^8, B_\mu\}$ basis is

$$\frac{1}{2}M^2 = \frac{1}{4}g^2 v_2^2 \begin{pmatrix} 3A & \frac{1}{\sqrt{3}}A & -2tA \\ \frac{1}{\sqrt{3}}A & \frac{1}{3}(3A+4) & \frac{2}{\sqrt{3}}t(A+2) \\ -2tA & \frac{2}{\sqrt{3}}t(A+2) & 4t^2(A+1) \end{pmatrix} \tag{12}$$

which is a singular matrix due to the vanishing eigenvalue associated to the photon mass. The nonvanishing eigenvalues, in the limit $A \to 0$, are

$$M_Z^2 = \frac{3}{2}g^2\frac{1 + 4t^2}{1 + 3t^2}v_1^2 \tag{13}$$

for the lighter bosons and

$$M_{Z'}^2 = \frac{2}{3}g^2(1 + 3t^2)v_2^2 \tag{14}$$

for the heavier neutral Hermitian gauge boson Z'. On the other hand from Eq. (8, the counterparts of charged non-Hermitian standard model gauge boson, have the following mass

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$$M_{W^\pm}^2 = \frac{3}{2} g^2 v_1^2 \tag{15}$$

so that in (331) gauge extension

$$\frac{M_Z^2}{M_{W^\pm}^2} = \frac{1 + 4t^2}{1 + 3t^2}. \tag{16}$$

Comparing with the standard model result,

$$\frac{M_Z^2}{M_{W^\pm}^2} = \frac{1}{1 - \sin^2 \theta_W}, \tag{17}$$

one obtains

$$t^2 = \frac{\sin^2 \theta_W}{1 - 4 \sin^2 \theta_W}. \tag{18}$$

Therefore the theory imposes an upper bound

$$\sin^2 \theta_W < \frac{1}{4} \tag{19}$$

with a Landau pole in $\sin^2 \theta_W = 1/4$. It is pertinent to point out here that the limiting condition $A \to 0$ used to obtain the constraint on the electroweak mixing angle θ_W in our paper implies a $v_\chi \equiv v_2$ on TeV scale for the choice $v_1 \sim 100$ GeV.

As such we are in fact dealing with three different mass scales represented by scalar expectation values, v_1(chosen to be ~ 100 GeV for establishing an upper limit on v_{σ_1}) related with $SU(3)_L \otimes U(1)_X$ symmetry breaking, $v_\chi \equiv v_2$(on TeV scale for the choice $v_1 \sim 100$ GeV), large enough to leave the new gauge bosons sufficiently heavy to keep consistency with low energy phenomenology and v_{σ_1} ($v_{\sigma_1} \leq 1.73$ GeV for $v_1 \sim 100$ GeV) much smaller than v_1 consistent with the experimental value of the ρ parameter. The choice of $v_{\sigma_1} = 0$ giving $\rho = 1$ has been used to obtain the mass matrix for the neutral gauge bosons in the $\{W_\mu^3, W_\mu^8, B_\mu\}$ basis in our paper.

We may also point out that although the existence of three scales is an important feature of the model at hand , it has no bearing on the double beta decay related features discussed in our paper.

ACKNOWLEDGMENTS

[1] F. Pisano and S. Shelly Sharma, Phys. Rev. D 57, 5670 (1998)

[2] R. Foot, O. F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D 47, 4158 (1993)

[3] J. C. Montero, C. A. de S. Pires, and V. Pleitez, Phys. Rev. D (1999)